INTRODUCTION TO NYSML

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INTRODUCTION TO NYSML

This presentation is dedicated to the memory of our colleague and co-author

Jan Siwanowicz

who suddenly passed away in March 2023 at the age of 46

SHORT HISTORY OF NYSML

- NYSML = New York State Mathematics League
- Founded by Alfred Kalfus who wanted teams of all-stars to gather for friendly face-to-face competition, which he thought would foster a mathematically positive environment for all involved
- Began in 1973, held “each” spring since then
- Cancelled in 2020
- Held virtually in 2021 and 2022

SHORT HISTORY OF NYSML

SHORT HISTORY OF NYSML

- Upstate/downstate site rotation and planning
- Teams usually represent a large (within NY state) geographic region (such as a county or several counties), but they can also represent an individual school or Math Circle
- One local league can be represented by several teams (four NYC teams, used to be six)
- DUSO team (Dutchess, Ulster, Sullivan, Orange counties)

SHORT HISTORY OF NYSML

- A team from Massachusetts asked to participate in the 1974 NYSML competition, and it took first place
- This led to the creation of the Atlantic Regions Mathematics League in 1976, which became the American Regions Mathematics League (ARML) in 1984
- The ARML competition is based on the format of the NYSML competition

SHORT HISTORY OF NYSML

- The NYC team has been the best team in the state every year the contest was held
- The next annual spring championship will be held on Saturday, April 13, 2024

NYSML FORMAT

- Teams consist of up to 15 members who are usually high-school students
- Problems are the same for all students
- Problems cover a wide variety of mathematical topics including algebra, geometry, number theory, combinatorics, probability, inequalities
- Calculus knowledge is not required but could be handy

NYSML FORMAT

- Calculators, phones, laptops, Internet-connected devices, etc. are banned
- Four main events that count toward overall team and individual results
- Team Round (short-answer based)
- Power Question (proof-based)
- Individual Round (short-answer based)
- Relay Round (short-answer based)

$a + b\sqrt{c}$

NYSML FORMAT

- Types of answers: a single number; a pair or a triple of numbers; a set or a list of numbers
- Form of answers: predefined (e.g. decimal (1.5), fraction (3/2), mixed number (1½), an expression with radicals), free form
- All answers must be simplified
- Answers 1 + 2, 6/4, √12 are incorrect, even if they could be simplified to the correct ones

NYSML FORMAT

- One event (Individual Round) allows contestants to compete for individual awards; others are only for team awards
- Combined individual scores are also included in the total team score
- The maximum number of points a team can earn is 300, up to 150 in Individual Round and up to 50 in each of the other three events

NYSML FORMAT

- Alternate contestants
- Reasons for bringing alternates
- Individual alternates
- Substitution and borrowing
- Incomplete teams
- Alternate teams
- Eligibility for awards

NYSML FORMAT

- Four main events, grading, an optional Tiebreaker Round, and the Award Ceremony are all done in a day

TEAM ROUND

- Teams of 15 students work collaboratively to solve ten problems in twenty minutes
- Problems are independent from each other
- Variety of topics and difficulty
- Each problem is worth 5 points, for a total of 50 points possible for the team
- Only answers (one team answer per problem) are scored

TEAM ROUND

- Grading is binary: the answer is either correct (5 points) or incorrect (0 points)
- To be counted as correct, the answer must be written in a proper form
- Grading is more or less straightforward, no special skills are required

TEAM ROUND

- Collaboration
- Topic preference
- Verification process
- Captain's role
- Seating arrangements
- Common strategies

TEAM ROUND

Problem #	Answer	Solved By	✔	✔✔
1	2024	SK	OK	GR
2				
3				
4				
5				
6				
7				
8				
9				
10				

POWER QUESTION

- Teams of 15 students work collaboratively to solve a multiple-part (usually ten or more) question around a central theme in one hour (guided mini-research)
- This is often an unusual, unique, or invented topic so students are forced to deal with complex new mathematical ideas, including definitions, examples, facts, relationships, hints, etc.

POWER QUESTION

- All written solutions provided by teams are scored
- Team solutions (not only answers!) must include answers, explanations and proofs with rigor, depending on the keywords used in question statements (compute, list, draw; determine, find, explain, show; prove, justify)
- Each problem is weighted (depending on its difficulty and the keyword used) for a possible total of 50 points

POWER QUESTION

- Even if not proved, earlier numbered items (only their statements!) may be used in solutions to later numbered items, but not vice versa
- Common lemmas
- Referencing the solution of one problem from the solution of another problem

POWER QUESTION

- Grading is not binary (partial progress in each item could be awarded some points based on the rubric)
- Grading is really time-consuming and requires special skills including an ability to read a terrible handwriting
- Explanations and proofs provided by students can vary significantly
- Some of them may be really different from the official solutions

POWER QUESTION

- Special procedure for grading multiple solutions for the same item provided by the same team

POWER QUESTION

- Collaboration
- Question type (keyword) preference
- Verification process
- Captain’s role
- Seating arrangements
- Common strategies
- Idea generators and solution writers

INDIVIDUAL ROUND

- Students answer ten questions in five pairs, taking ten minutes for each pair
- There is no collaboration
- Problems are independent from each other
- Variety of topics and difficulty
- Each problem is worth 1 point per contestant, for a total of 10 points possible for the contestant, and a grand total of 150 points possible for the team

INDIVIDUAL ROUND

- Only answers (one contestant answer per problem) are scored
- Grading is binary: the answer is either correct (1 point) or incorrect (0 points)
- To be counted as correct, the answer must be written in a proper form
- Grading is more or less straightforward, no special skills are required

INDIVIDUAL ROUND

- Correct answers are announced after collecting student answer sheets for each problem pair, so contestants can easily keep track of their scores
- Possible strategies

RELAY ROUND

RELAY ROUND

- Teams are broken into five groups of three (relay teams) if possible
- Each relay team tries to answer a string of questions, where the answer to the first question is needed to solve the second, and the answer to the second question is needed to solve the third

RELAY ROUND

- Within each relay team, the first team member solves a problem and passes the answer to the next team member, who plugs that answer into their question, and so on
- Only answers (one final answer per relay team submitted by the third team member) are scored

RELAY ROUND

- The allotted time is six minutes, but extra points are given for solving the problem in three minutes and not providing ANY answer in six minutes
- Solving the relay in three minutes gives 5 points (per relay team), solving it in six minutes gives 3 points, for a sub-total of 25 points possible for the team

RELAY ROUND

- The whole process is done twice (with different strings of problems), for a total of 50 points possible for the team

RELAY ROUND

- Grading is binary: the answer is either correct (5 or 3 points depending on the time the final answer was provided) or incorrect (0 points)
- To be counted as correct, the answer must be written in a proper form
- Grading is more or less straightforward, no special skills are required

RELAY ROUND

- Communication
- Changing the answer
- Confirming the answer
- Communicating uncertainty
- Communicating info about an answer
- Common strategies
- Special strategies

RELAY ROUND

- Ability to work quickly and accurately under pressure
- Ability to communicate effectively with teammates
- Selecting roles (#1, #2, #3) within a relay team

TIEBREAKER ROUND

- To break, or not to break, that is the question
- Top four students are recognized
- Examples of ties in individual scores:
- 10, 10, 9, 9, 9
- 10, 9, 9, 8, 8, 8, 8, 8, 8, 8, 8, 8, 8
- 50 students with score 10
- Virtually impossible case: 10, 9, 8, 7, 6, 6, 6

TIEBREAKER ROUND

- All students are listed based on their individual scores in descending order
- The score of the person #4 from the top is the passing score
- All students with their scores at least as high as the passing score are invited to the Tiebreaker Round, even if such students do not have any score ties

TIEBREAKER ROUND

- In the Tiebreaker Round, students with high Individual Round scores come to the front of the auditorium and answer questions one at a time, using their times to break ties (for individual scores only) and award final prizes
- The goal is to solve TB questions correctly, but time also matters

TIEBREAKER ROUND

- Some ties can be broken (resolved) after the first TB problem (TB-1), some other ties can be broken after TB-2, etc.
- Usually there are up to 3 TB problems prepared for the competition, but in some cases additional TB questions may be required
- Ability to work quickly and accurately under significant pressure

TEAM TIEBREAKER PROCEDURE

- Team score ties are broken (resolved) without an additional round, by first considering the sum of the Team and Power Question Rounds, then the Relay Round total

INDIVIDUAL AWARDS

- Top four students (based on their individual scores and the results of the Tiebreaker Round) are recognized
- The Individual Champion earns the Curt Boddie Award in memory of Curt, who was NYSML's President for many years
- Individual High Scorer Award is awarded to all other participants in the Tiebreaker Round

INDIVIDUAL AWARDS

- Team High Scorer Medal is awarded to all students with the highest scores in their teams (excluding scores of all students who participated in the Tiebreaker Round)
- Every team gets at least one individual award

TEAM AWARDS

- Top three teams in Division A are recognized
- Top three teams in Division B are recognized

DOMINANT TEAMS

- Teams with consistently high results year over year
- Unofficial Team Ranking at IMO
- Team China (since 1986): mostly in top 2, with few exceptions (#4, #8, #6, #3, #3)
- Team USA (since 1974): mostly in top 5, with few exceptions (#6, #7, #11, #10, #6)
- Team Ukraine (since 1993): mostly in top 20, few times in top 10, once in top 4

IMAGINARY IMO RULE CHANGE

- Imagine that IMO Board allows LARGE provinces/states/territories (with a total population of 10 million or more) within a country to be represented at IMO by their own teams, in addition to the country team

IMAGINARY IMO RULE CHANGE

- China: 22 provinces (excluding Taiwan), 5 autonomous regions, 4 municipalities, 2 Special Administrative Regions
- USA: 50 states, 1 federal district, 5 major territories
- Ukraine: 27 regions (including 2 cities with special status and 1 autonomous republic)

IMAGINARY IMO RULE CHANGE

- China: 21 LARGE provinces (excluding Taiwan), 3 LARGE autonomous regions, 4 LARGE municipalities, 0 LARGE Special Administrative Regions
- USA: 10 LARGE states, 0 LARGE federal districts, 0 LARGE major territories
- Ukraine: 0 LARGE regions

IMAGINARY IMO RULE CHANGE

- China: 1 + 28 = 29 teams
- USA: 1 + 10 = 11 teams
- Other countries with LARGE provinces/states/territories (e.g. India)
- Ukraine: 1 team

IMAGINARY IMO RULE CHANGE

- How many IMO teams from China (out of 29) would appear in top 10 (20, 30, 40, 50), on average?
- How many IMO teams from USA (out of 11) would appear in top 10 (20, 30, 40, 50), on average?
- What are the chances for the only IMO team from Ukraine to appear in top 10 (20, 30, 40, 50)?

BACK TO NYSML

- NYC is usually represented at NYSML by several teams (between 4 and 6)
- In the past, it was not uncommon for NYC teams to get all three team awards in Division A
- NYSML changed Team Award rules to allow not more than one team award per year per member league

NYSML 2015: TEAM ROUND

T-1. For two positive numbers x and y, we define their arithmetic mean as $\frac{x+y}{2}$, their geometric mean as $\sqrt{xy}$, and their harmonic mean as $\frac{2}{\frac{1}{x}+\frac{1}{y}}$. Suppose that two positive numbers have a geometric mean of 24 and a harmonic mean of 22. Compute their arithmetic mean.

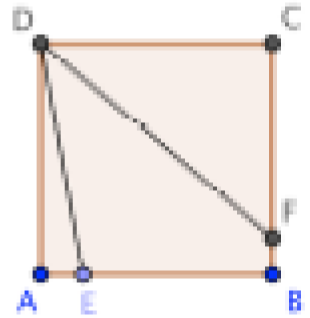
T-2. If $(x+2z):(2y+z):(2x+y) = 1:3:5$ and $x+y+z=18$, compute the value of z.

T-3. There are 52 balls in a box. Each ball has a number. Four of the balls are numbered 0, four are numbered 1, and so on, such that the highest number on a ball is 12 (and this occurs for four balls). Three balls are chosen from the box without replacement. Compute the probability that at least one ball will have a two-digit number.

T-4. The perimeter of regular dodecagon $DISCOUNTABLE$ is 60. Compute its area in the form $a+b\sqrt{c}$, where a, b, and c are integers, and c cannot be divided by the square of any prime.

NYSML 2015: INDIVIDUAL ROUND

I-8. In square $ABCD$, E is on $\overline{AB}$ and F is on $\overline{BC}$ such that $\overline{DF}$ is an angle bisector of $\angle EDC$. Given that $DE = 20$ and $AD = 15$, compute $AE + CF$.



I-9. Consider a sequence $\{n_i\}$ for which $n_1 = 2$, $n_2 = 0$, $n_3 = 1$, $n_4 = 5$, $n_5 = 20$, $n_6 = 15$, and $n_i = n_{i-1} - n_{i-2} + n_{i-3} - n_{i-4} + n_{i-5} - n_{i-6}$ for $i \geq 7$. Compute n_{2015}.

I-10. Compute the number of positive integers n such that $n \leq 2015$ and n is divisible by $\lfloor \sqrt{n} \rfloor$, which is the greatest integer not exceeding $\sqrt{n}$.

NYSML 2015: RELAY ROUND

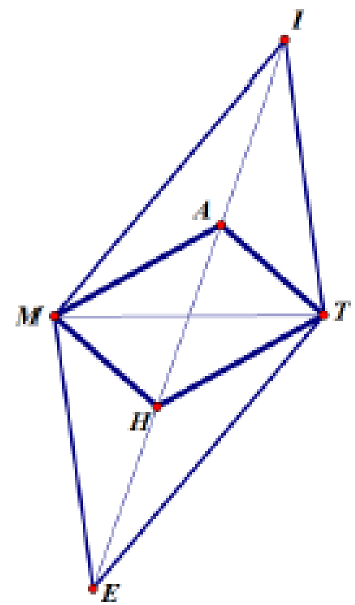
R1-1. Compute the two-digit number that is equal to one more than three times the sum of its digits.

R1-2. *Let N be the two-digit prime you will receive.* The four-digit number $Y = \underline{2}\,\underline{0\,P\,Q}$ is divisible by N. Compute the number of distinct possible values of Y.

R1-3. *Let N be the number you will receive.* The solutions of $x^3 - 4x^2 + 6x - N = 0$ are p, q, and r. Compute the value of $(p+q)(p+r)(q+r)$.

NYSML 2015: TIEBREAKER ROUND

TB-1. In parallelogram $MATH$, $MA = 11$ and $AT = 9$. In parallelogram $TIME$, $TI = 13$ and $MI = 17$. Vertices A and H of $MATH$ trisect diagonal $\overline{IE}$ of $TIME$. Compute the length IE.



TB-2. The three-digit octal (base-8) number $N = \underline{A}\,\underline{B}\,\underline{C}$ is 5 times the two-digit octal number $\underline{A}\,\underline{C}$. Compute the greatest possible value of N, giving your answer in base 8.

NYSML 2015: POWER QUESTION

Power Question 2015: Irregular Regular Polygons

The Regulars:

Recall that for any integer $n \geq 3$ there exists a regular polygon having n sides with all sides congruent and all internal angles congruent. For the purpose of this question, we assume that all sides have length 1, making this (convex) polygon R_n unique for each $n \geq 3$.

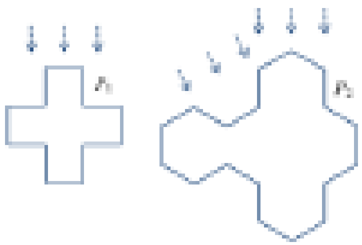
For any regular polygon R we define $\alpha(R)$ as the degree-measure of any internal angle of R.

P-1. a. Compute the areas of R_3, R_4, R_6, and R_8. [4 pts]

b. Provide an explicit formula (in terms of n) for $\alpha(R_n)$. [1 pt]

The Irregulars:

Consider the polygons P_1 and P_2 below. All of their sides have length 1, and for each of these polygons all of the non-reflex angles are congruent, but some of these angles are internal and some others are not. P_1 and P_2 are examples of **Irregular Regular Polygons** (IRPs).



NYSML 2015: POWER QUESTION

Before we formally define an IRP, let's consider any polygon P. As usual, we do not allow self-intersecting polygons or polygons with overlapping vertices, but we do allow non-convex polygons.

Each vertex of P and two sides of P sharing this vertex form two angles. One of them is an internal angle of P and the other one is the corresponding explementary angle. Note that exactly one of these two angles is a reflex angle. Therefore, any polygon P with n vertices has n pairs of explementary angles, or $2n$ angles altogether – n reflex and n non-reflex ones.

A regular polygon could be defined as a convex polygon with all sides congruent and all non-reflex angles congruent. Now, we define an **IRP** as a non-convex polygon with all sides congruent and all non-reflex angles congruent. We continue to assume that all sides have length 1. Note that regular polygons R_n, because they are convex, are not IRPs.

For any regular polygon R we could define $\alpha(R)$ as the degree-measure of any non-reflex angle

NYSML 2015: POWER QUESTION

of R. Similarly, if P is an IRP, we define $\alpha(P)$ as the degree-measure of any non-reflex angle of P, the IRP.

P-2. a. Compute the perimeters and areas of P_1 and P_2. [4 pts]

b. Compute the least possible radius of a disk which fully covers P_2. [1 pt]

P-3. a. Show that for every IRP P there exists an integer $n \geq 3$ such that $\alpha(P) = \alpha(R_n)$. [3 pts]

b. Show that every IRP has at least four internal non-reflex angles. [2 pts]

The Families:

For any integer $n \geq 3$ we define **IRP-n** as the set (family) of all IRPs P such that $\alpha(P) = \alpha(R_n)$. The result of **P3-a** means that every IRP belongs to exactly one of these families. In the examples above, $P_1 \in$ IRP-4 and $P_2 \in$ IRP-6.

Hint: To solve some of the problems below, it might be useful to take a look at an IRP along a line parallel or perpendicular to one of the IRP's sides.

P-4. a. Show that the family IRP-3 is empty. [1 pt]

b. Draw two IRPs, $P_3 \in$ IRP-4 and $P_4 \in$ IRP-6, which have neither a line of symmetry nor a center of symmetry. [2 pts]

c. Draw two non-congruent IRPs, P_5 and P_6, having the same perimeters and the same areas. [2 pts]

NYSML 2015: POWER QUESTION

P-5. a. Prove that every IRP from IRP-n has n more internal non-reflex angles than internal reflex angles. [2 pts]

b. Show that the perimeter of every IRP from IRP-n has the same parity as n. [1 pt]

c. Show that for every even integer $p \geq 10$, there exists an IRP from IRP-6 with perimeter p. [2 pts]

P-6. a. Show that the perimeter of every IRP from IRP-4 is at least 12. [1 pt]

b. Show that the perimeter of every IRP from IRP-n is at least $n + 2$. [1 pt]

c. Show that the perimeter of every IRP from IRP-n is at least $n + 4$. [3 pts]

P-7. a. Show that $2\alpha(R_5) + \alpha(R_{10}) = 360°$. [1 pt]

b. Draw an IRP from IRP-5 with perimeter 25. [2 pts]

c. Draw an IRP from IRP-5 with perimeter not equal to 25. [2 pts]

P-8. a. Prove that the perimeter of every IRP from IRP-5 is a multiple of 5. [3 pts]

b. Draw the unique IRP with the least possible perimeter. [2 pts]

NYSML 2015: POWER QUESTION

P-9. a. Show that the family IRP-6 contains infinitely many different (non-congruent) IRPs. [1 pt]

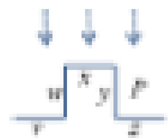
b. Show that the family IRP-5 contains infinitely many different (non-congruent) IRPs. [2 pts]

c. Show that for every integer $n \geq 4$, the family IRP-n contains infinitely many different (non-congruent) IRPs. [2 pts]

P-10. Prove that there exists an IRP with a prime perimeter. [5 pts]

NYSML 2015: POWER QUESTION

P-6. a. Let P be an arbitrary IRP from IRP-4. Assume that P has only horizontal and vertical sides. Let x be one of the top horizontal sides of P. Two neighboring sides, w and y, are vertical, and their neighboring sides, v and z, are horizontal. Neither of these two horizontal sides appear directly below x to avoid a self-intersection.

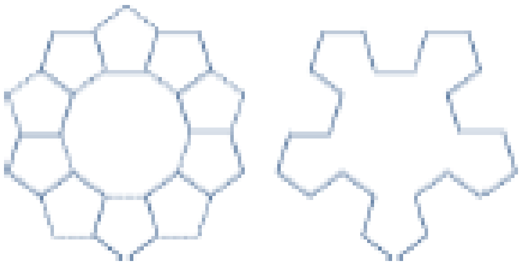


So if one looks at P from above along a vertical line, one will see at least three different horizontal sides (all of them differ from v, x, and z) not blocked by other sides. This means that P has at least six different horizontal sides. Similarly, P has at least six different vertical sides, and therefore its perimeter is at least 12.

NYSML 2015: POWER QUESTION

P-7. a. The answer to **P1-b** implies that $\alpha(R_5) = 108°$ and $\alpha(R_{10}) = 144°$. Therefore, $2\alpha(R_5) + \alpha(R_{10}) = 360°$.

b. Draw R_{10} and then on each of its sides place an instance of R_5 (externally). The result of **P7-a** implies that each of these ten regular pentagons will share a side with two neighboring regular pentagons. Now it is straightforward to highlight some of their sides to get a required (flower-like) IRP from IRP-5 with perimeter 25 (at the right in the figure below).



Alternatively, we can apply the vector-based method described in the solution to **P7-c** to get another IRP from IRP-5 with perimeter 25 (shown below).

NYSML 2015: POWER QUESTION

- Selected and approved for NYSML 2016
- Shifted to NYSML 2015
- Why?

LIVE RELAY ROUND SIMULATION

R1. Let the sequence $\{T_n\}$ be the sequence of triangular numbers, $T_1 = 1, T_n = T_{n-1} + n, n \geq 2$.
Let the sequence $\{F_n\}$ be the Fibonacci sequence, $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}, n \geq 3$.
Compute the product of all common elements in both sequences that are less than 2024.

R1. Нехай послідовність $\{T_n\}$ є послідовністю трикутних чисел, $T_1 = 1, T_n = T_{n-1} + n, n \geq 2$.
Нехай послідовність $\{F_n\}$ є послідовністю Фібоначчі, $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}, n \geq 3$.
Обчисліть добуток усіх спільних елементів в обох послідовностях, менших за 2024.

LIVE RELAY ROUND SIMULATION

R2. *Let N be the number you will receive.* Compute the least prime that doesn't divide $\frac{N^N-1}{N-1}$.

R2. *Нехай N буде числом, яке ви отримаєте*. Обчисліть найменше просте число, на яке не ділиться $\frac{N^N-1}{N-1}$.

LIVE RELAY ROUND SIMULATION

R3. *Let r be the even number you will receive.* Compute the greatest possible perimeter of a right triangle with integer side lengths and inradius r.

R3. *Нехай r буде парним числом, яке ви отримаєте*. Обчисліть найбільший можливий периметр прямокутного трикутника з цілими довжинами сторін і радіусом вписаного кола r.

LIVE RELAY ROUND SOLUTIONS

R1. Let the sequence $\{T_n\}$ be the sequence of triangular numbers, $T_1 = 1, T_n = T_{n-1} + n, n \geq 2$. Let the sequence $\{F_n\}$ be the Fibonacci sequence, $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}, n \geq 3$. Compute the product of all common elements in both sequences that are less than 2024.

Solution. $T_n = \frac{n(n+1)}{2}, n \geq 1$; $k = T_n \leftrightarrow 8k + 1 = 4n^2 + 4n + 1 = (2n+1)^2$.

$8k + 1 = m^2 \rightarrow 8k + 1 \equiv 0, 1 \ (mod\ 3) \leftrightarrow 8k \equiv 2{,}0 \ (mod\ 3) \leftrightarrow -k \equiv 0{,}2 \ (mod\ 3)$
$\leftrightarrow k \equiv 0{,}1 (mod\ 3)$;

$8k + 1 = m^2 \rightarrow 8k + 1 \equiv 0, 1{,}4 \ (mod\ 5) \leftrightarrow 8k \equiv 4{,}0{,}3 \ (mod\ 5) \leftrightarrow -2k \equiv 4{,}0{,}3 \ (mod\ 5)$
$\leftrightarrow -4k \equiv 8{,}0{,}6 \ (mod\ 5) \leftrightarrow k \equiv 0{,}1{,}3 (mod\ 5)$.

The elements of the Fibonacci sequence that are less than 2024 are:
1, 1, 2, 3, 5 ,8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597.

$8 \cdot 610 + 1 = 4881 = 70^2 - 19 \in (69^2, 70^2)$; $8 \cdot 13 + 1 = 105 \in (10^2, 11^2)$.

Numbers $1 = T_1, 3 = T_2, 21 = T_6, 55 = T_{10}$ are indeed triangular numbers, so the answer is
$1 \cdot 1 \cdot 3 \cdot 21 \cdot 55 = \mathbf{3465}$.

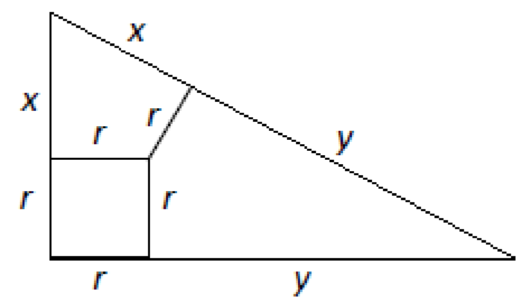
LIVE RELAY ROUND SOLUTIONS

R2. *Let N be the number you will receive.* Compute the least prime that doesn't divide $\frac{N^N-1}{N-1}$.

Solution. $\frac{N^N-1}{N-1} = N^{N-1} + N^{N-2} + \cdots + N^2 + N + 1$ (N terms). Since $N = 3465$ is odd, each term is odd, the number of terms is odd, so their sum is odd, and the answer is **2**.

LIVE RELAY ROUND SOLUTIONS

R3. *Let r be the even number you will receive.* Compute the greatest possible perimeter of a right triangle with integer side lengths and inradius r.



Solution. WLOG we can assume that $x \le y$. Note that $x, y \in N$.

$(r+x)^2 + (r+y)^2 = (x+y)^2 \leftrightarrow r^2 + rx + ry = xy$

$\leftrightarrow (x-r)(y-r) = 2r^2$. Plugging in $r = 2$, we can see that only the following cases are possible:

$x - 2 = 1, y - 2 = 8, x = 3, y = 10, r + x = 5, r + y = 12, x + y = 13$, (5, 12, 13) is indeed a right triangle with integer side lengths, inradius 2, and perimeter 30;

$x - 2 = 2, y - 2 = 4, x = 4, y = 6, r + x = 6, r + y = 8, x + y = 10$, (6, 8, 10) is indeed a right triangle with integer side lengths, inradius 2, and perimeter 24.

So the answer is $\max(30, 24) = \mathbf{30}$.

LIVE RELAY ROUND DISCUSSION

- Common strategies for Relay Team members
- Additional smart strategy for Relay Team member #1
- Additional smart strategies for Relay Team member #2
- Additional smart strategy for Relay Team member #3

POSSIBLE APPLICATIONS

- Using general relays and topic-focused relays
- Asking students to prepare their own relays
- Using student-prepared relays with different student groups
- Using Power Question topics for student research activities (for each Power Question, there are many related problems that for various reasons were not included in the event)

REFERENCES

- George Reuter, Michael Curry. **Problems to Enrich and Challenge: NYSML - New York State Mathematics League Contests 2011 - 2019** / USA: "ARML", 2021
- https://en.wikipedia.org/wiki/New_York_State_Mathematics_League
- https://nysml.com